Lecture notes

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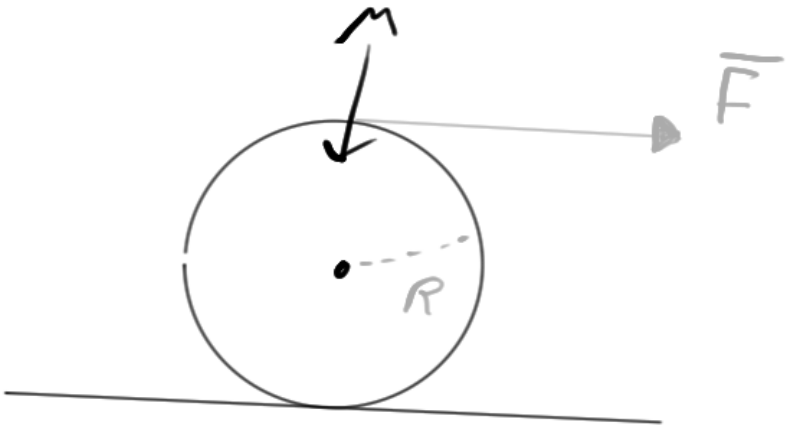
Repetion

$$\sum \bar{F}^{ext} = Ma_{\bar{CM}} \qquad \sum \bar{\tau}^{ext} = I\bar{\alpha} \qquad (1)$$

$$\sum \bar{F}^{ext} = \frac{d\bar{P}}{dt} \qquad \sum \bar{\tau}^{ext} = \frac{d\bar{L}}{dt} \qquad (2)$$

Exempel 1



$\bar{F} = F\hat{i},\ \bar{f} = f\hat{i}$

Använd 1: $\bar{F} + \bar{f} = Ma_{\bar{CM}}$

$F\hat{i} + f\hat{i} = Ma_{CM}\hat{i} \rightarrow F + f = Ma_{CM}$

$\sum \bar{\tau} = (R\hat{j} \times F\hat{i}) + (-R\hat{j} \times f\hat{i}) = RF(-\hat{k}) + Rf(+\hat{k}) = I\alpha\hat{k}$

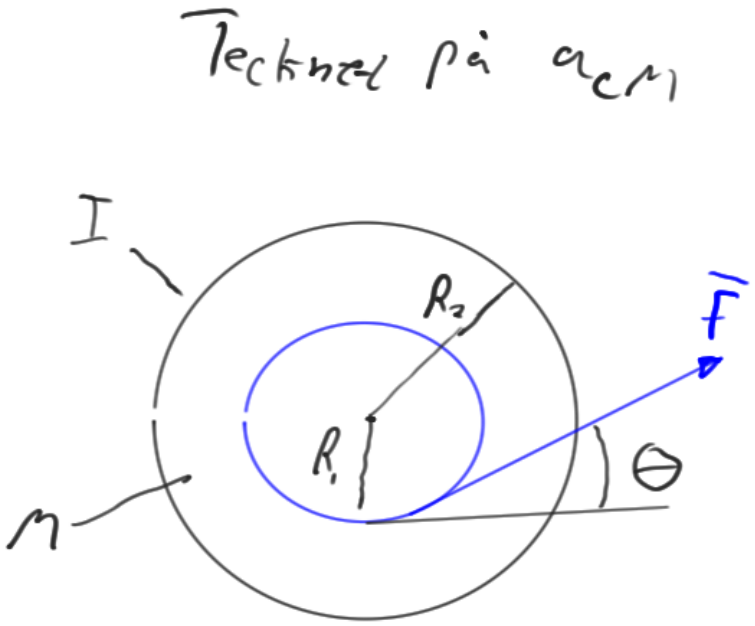
$\rightarrow \underline{Rf - RF = \frac{1}{2}MR^2\alpha}$

$|a_{\bar{CM}}| = R|\bar{\alpha}| \rightarrow a_{CM} = -R\alpha \rightarrow \alpha = -\frac{a_{CM}}{R} \rightarrow 2f - 2F = -Ma_{CM}$

$2f - 2F = -Ma_{CM}$ addera med $F + f = Ma_{CM} \rightarrow -F + 3f = 0 \rightarrow f = \frac{F}{3}$

Friktionen är riktad åt samma håll som krafen $\bar{F}$

Exempel 2



$$\bar{f} = f\hat{i}$$

$$F\cos\theta + f = Ma_{cm}$$

$$-R_2F\cos\theta - R_2f = -MR_2a_{CM}$$

$$R_1F(+\hat{k}) + R_2f(+\hat{k}) = I\alpha\hat{k} = -I\frac{a_{CM}}{R_2}\hat{k}$$

$$R_1F + R_2f = -I\frac{a_{CM}}{R_2}$$

Addera ekvationerna:

$$-R_2F\cos\theta \qquad -R_2f = -Ma_{CM}R_2$$

$$R_1F \qquad +R_2f = -I\frac{a_{CM}}{R_2}$$

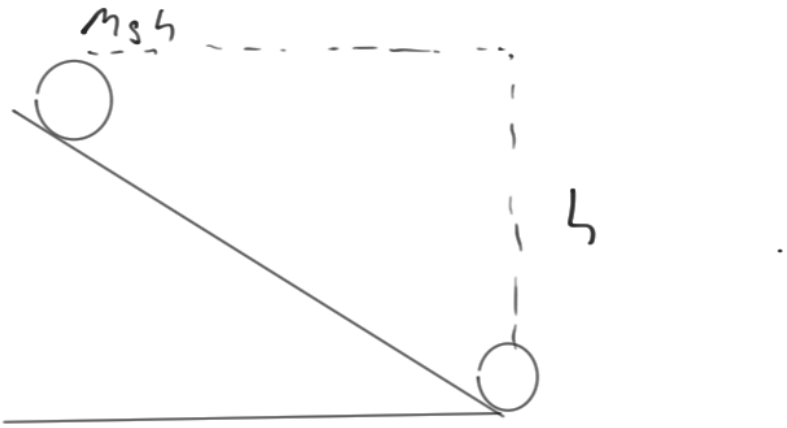
$$-R_2F\cos\theta \qquad +R_1F = -\left[R_2M + \frac{I}{R_2}\right]a_{CM}$$

$$\rightarrow a_{CM} = \frac{F}{R_2M + \frac{I}{R_2}}R_2\left(cos\theta - \frac{R_1}{R_2}\right)$$

Exempel 3

Burk i backe.

$$K = K_R + K_T = \frac{1}{2}I\omega^2 + \frac{1}{2}MV_{CM}^2$$



$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}MV_{CM}^2 = \frac{1}{2}I\frac{V_{CM}^2}{R^2} + \frac{1}{2}MV_{CM}^2 = \frac{1}{2}V_{CM}^2\left[\frac{I}{R^2} + M\right]$$

$$\rightarrow V_{CM}^2 = \frac{2Mgh}{\frac{I}{R^2} + M}$$

Två burkar:

$$I_{full} \sim \frac{1}{2}MR^2 \rightarrow \quad \frac{I}{R^2} + M = \frac{\frac{1}{2}MR^2}{R^2} + M = \frac{3}{2}M,\ V_{CM}^2 = \frac{2gh}{\frac{3}{2}} = \frac{4}{3}gh$$

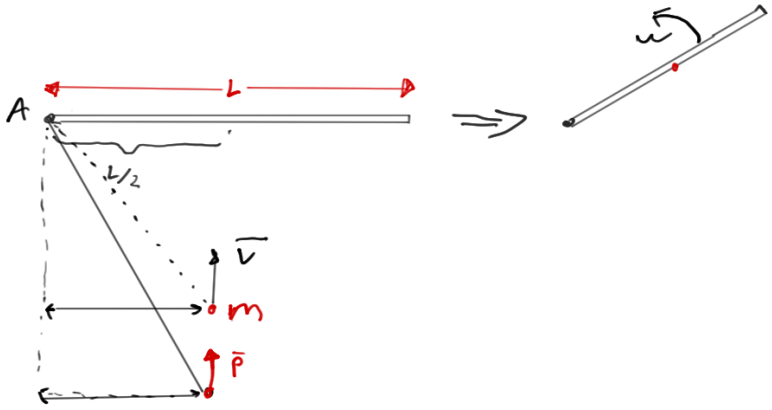
$$I_{tom} \sim \frac{1}{2}mR^2 \rightarrow \qquad \frac{I}{R^2} + m = \frac{mR^2}{R^2} + m = 2m,\ V_{CM}^2 = \frac{2gh}{2} = gh$$

Exempel 4

Lerklump kastad mot dörr. Mekansik energi och Rörelsemäng bevaras inte. Inga yttre vridande moment m.a.p A, Alltså bevaras $\bar{L}$.

$\bar{L} = \bar{r} \times \bar{p}$

$\bar{L} = I\bar{\omega}$



$$L_i = \frac{l}{2}mv \qquad L_f = \left[\frac{1}{3}ML^2 + m\left(\frac{l}{2}\right)^2\right]\omega \rightarrow \omega = \frac{\frac{l}{2}mv}{\frac{1}{3}Ml^2 + m\left(\frac{l}{2}\right)^2}$$