Lecture notes

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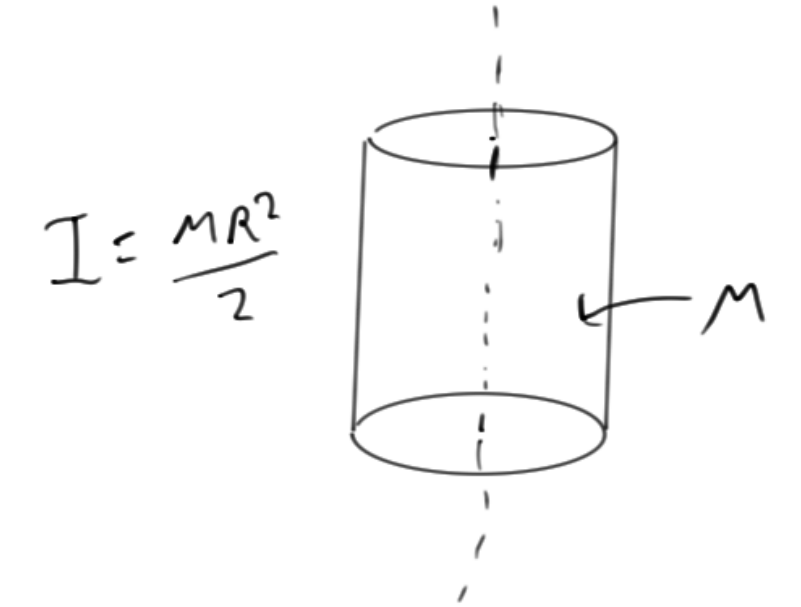
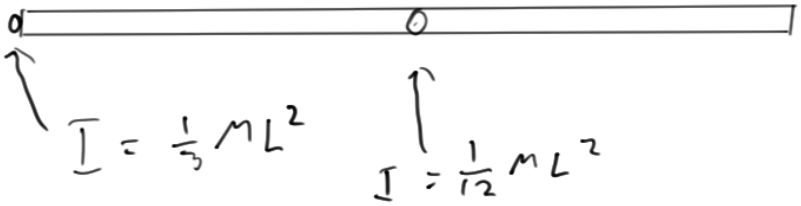
Rotations rörelser





Tröghetsmoment

$$I = \int_{hela} r^2 dm$$

Vridandemoment

$$\tau \equiv \bar{r} \times \bar{F}$$

Rörelsemängdsmoment $\bar{L}$

$$\bar{L} \equiv \bar{r} \times \bar{p},\ \bar{p} = m\bar{v}$$
$$\bar{F} = \frac{d\bar{p}}{dt},\ \bar{\tau} = \frac{d\bar{L}}{dt}$$

Partikel mekanik	Rotation
m	I
x	θ
v	ω
a	α
F	τ
P	L
$\frac{mV^2}{2}$	$\frac{i\omega^2}{2}$

Bevarade storeheter

Mekanisk energi	Se upp!
Rörelsemängd	Inga externa krafter
Rörelsemängdsmomenet	

Härledning av former

Gäller $\bar{\tau} = I\bar{\alpha}$? Sambandet mellan rotation och partikel fysik antyder det.

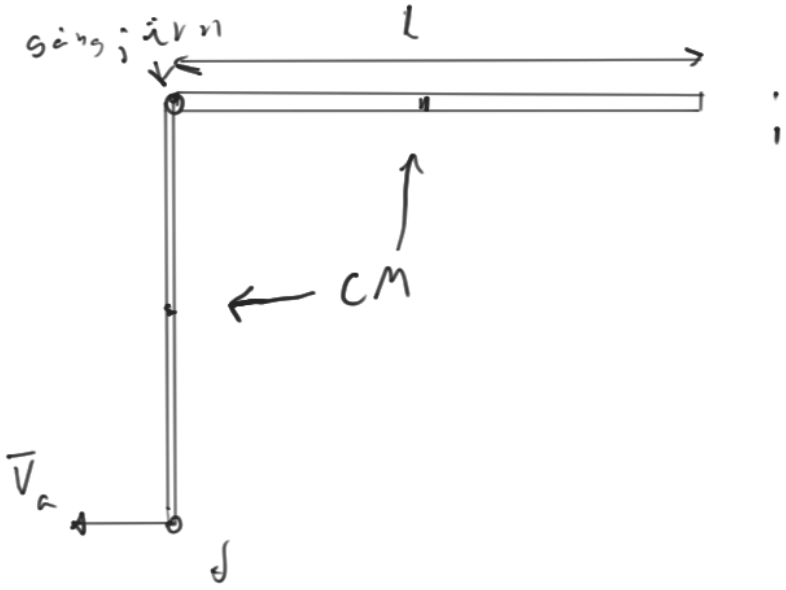
$$\bar{L} = \bar{r} \times \bar{p} = \bar{r} \times m\bar{v}$$
$$d\bar{L} = \bar{r} \times dm\bar{v} = dm(\bar{r} \times \bar{v})$$
$$|d\bar{L}| = dmrv = dmr(\omega r) = \omega r^2 dm$$
$$|\bar{L}| = \int_{hela} |d\bar{L}| = \omega \int r^2 dm \rightarrow a|\bar{L}| = I\omega$$
$$\tau = \frac{dL}{dt}, L = I\omega \rightarrow \tau = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha$$

Från partikel mekaniken $\sum_i \bar{F}_i^{ext} = M\bar{a}_{CM}$

Exempel

Exempel 1

Den mekaniska energin bevaras.



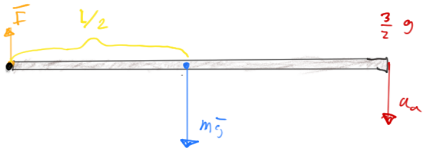
$\underline{v = \omega r}$

$K_i = 0$ $\qquad v_i = mgl\frac{1}{2}$

$L_f = \frac{1}{2}I\omega^2$ $\qquad v_f = 0$

$$mgl\frac{1}{2} = \frac{1}{2}\cdot\frac{1}{3}ml^2\omega^2 = mgl\frac{1}{2} = \frac{1}{2}\cdot\frac{1}{3}ml^2(\frac{v_a}{l})^2 v_a = \sqrt{3gl}$$

Acceleration av masscentrum:

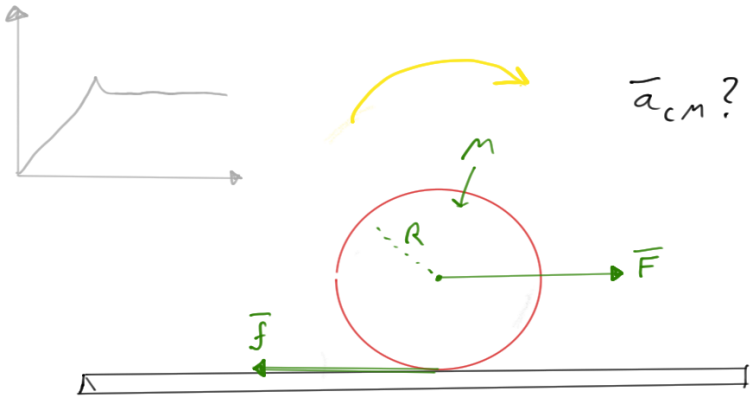


$$I = \frac{1}{3}ml^2,\ \tau = \frac{lmg}{2}$$

$$\frac{1}{3}ml^2 = \frac{1}{3}ml^2\alpha,\ \alpha = \frac{a_{CM}}{\frac{l}{2}} \rightarrow a_{CM} = \frac{3}{4}g$$

$$mg - F = m\frac{3}{4}g$$

Exempel 2



$$\bar{F} + \bar{f} = M\bar{a_{CM}}$$
$$fR = I\alpha$$

$$I = \frac{1}{2}MR^2,\ a_{CM} = \alpha R$$
$$fR = \frac{1}{2}MR^2\frac{a_{CM}}{R} \rightarrow f = \frac{1}{2}Ma_{CM} \rightarrow a_{CM} = \frac{2f}{M}$$
$$F - f = M\frac{2f}{M} \rightarrow f = \frac{F}{3}$$

Exempel 3