Lecture notes

Lukas Rahmn

Tenta räkning,

Del A, 5/10, Del B, 7/15

Del A

Uppgift A1

$$x(t) = cos(300t), h(t) = 600e^{-\sqrt{3}\cdot 100t}u(t)$$

Bestäm $y = h * x$, (1p)

Lösning:

$$x(t) = \frac{1}{2}(e^{j300t} + e^{-j300t})$$
$$y(t) = \frac{1}{2}H(j300)e^{j300t} + \frac{1}{2}H(-j300)e^{-j300t}$$
$$H(j\omega) = 600\frac{1}{\sqrt{3}\cdot 100 + j\omega} \rightarrow, \textit{(Tabell värde)}$$
$$y(t) = \frac{1}{2}600 \cdot \frac{1}{\sqrt{3}\cdot 100 + j300}e^{j300t} + \frac{1}{2}600\frac{1}{\sqrt{3}\cdot 100 - j300} \cdot e^{-j300t}$$

Uppgift A2

Bestäm (c_k) om $x(t) = 5 + 2 \cdot \cos(500t + \pi/6)$ Lösning:

$[T = \frac{2\pi}{500} = \frac{\pi}{250}]$

$$x(t) = 5 + 2 \cdot \frac{1}{2}(e^{j(500t+\pi/6)} + e^{-j(500t+\pi/6)})$$
$$= e^{-j\pi/6} \cdot e^{-j500t} + 5 \cdot \{e^{j\omega_0 t} = 0\} + e^{+e^{j\pi/6}e^{j500t}}$$
$$\begin{cases} c_1(x) = e^{-j\pi/6} \\ c_0(x) = 5 \\ c_1(x) = e^{j\pi/6} \\ c_k(x) = 0, k \neq 0, \pm 1 \end{cases}$$

24 Aug /16

Uppgift 4

$$T = 2\pi \cdot 10^{-2} s$$
$$x(t) = \begin{cases} 1, 0 \le t < T/2 \\ -1, T/2 \le t < T \end{cases}$$
$$x(t+T) = x(t), \forall t$$
$$\text{Lågpassfilter:} y = h * x, H(j\omega) = \begin{cases} 1, |\omega| < 420 \text{ rad/20} \end{cases}$$

Jämför medelenergierna för x och y. Lösning:

$E_T = \frac{1}{T}\int_0^T |x(t)|^2 dt$

$$E_T(x) = \frac{1}{T}\int_0^T |x(t)|^2 dt = 1$$
$$E_T(y) = \frac{1}{T}\int_0^T |y(t)|^2 dt =$$
$$\{\text{Parseval}\} = \sum_{k=-\infty}^{\infty} |c_k(y)|^2 = \{c_k(y) = H(j\omega_k)c_k(x), \omega_k = \frac{2\pi k}{T}\} =$$
$$= \sum_{k=-\infty}^{\infty} |H(j\omega_k)|^2 \cdot |c_k(x)|^2 \neq \underline{\text{omm}} |w_k| = \frac{2\pi|k|}{T} < 420 \iff$$
$$|k| < \frac{420T}{2\pi} = 4.2 \iff k = 0, \pm1, \pm2, \pm3, \pm4$$
$$= \sum_{k=-4}^{4} |c_k(x)|^2$$
$$c_x(x) = \frac{1}{T}\int_0^T x(t)e^{-j\omega_k t}dt = \frac{1}{T}\left(\int_0^{T/2} e^{-j\omega_k t}dt - \int_{T/2}^{T} e^{-j\omega_k t}dt\right)$$
$$= \frac{1}{T}\left(\left[\frac{-e^{-j\omega_k t}}{j\omega_k}\right]_{t=0}^{T/2} - \left[\frac{e^{-\omega_k t}}{j\omega_k}\right]_{t=T/2}^{T}\right)$$
$$= \frac{1}{j\omega_k T}\left(-e^{-j\omega_k T/2} + 1 - \left(-e^{-j\omega_k T} + e^{-j\omega_k T/2}\right)\right) =$$
$$= \left\{\omega_k T/2 = \frac{2\pi k}{T}T/2 = \pi k, \omega_k T = 2\pi k, e^{-j\omega_k T} = 1, e^{-j\omega_k T/2} = (-1)^k\right\}$$
$$= \{k \neq 0\} = \frac{1}{2\pi jk}(2 - 2\cdot(-1)^k) = \begin{cases} 0, k = 2n, n \neq 0 \\ \frac{2}{\pi jk}, k = 2n+1 \end{cases}$$

$c_0(x) = \frac{1}{T}\int_0^T x(t)dt = 0 \rightarrow$

$\begin{cases} 0, k = 2n, n \neq 0 \\ \frac{2}{\pi jk}, k = 2n+1 \end{cases}$

$\rightarrow E_T(y) = \sum k = -4^4 |c_k(x)|^2 = |c_{-1}(x)|^2 + |c_1(x)|^2 + |c_{-3}(x)|^2 + |c_3(x)|^2 =$

$2 \cdot \frac{4}{\pi^2} + 2 \cdot \frac{4}{9\pi^2} = \frac{80}{9\pi^2} < 1$

31 okt/14

Uppgift 5

LTI-system $y = h * x$, $h(t) = \delta(t) + (\cos t + 2\sin t)u(t), x(t) = e^{-2t}u(t)$

Bestäm y, (5p)

Lösning:

Angående δ: $(\delta * z)(t) = z(t), \forall t, \forall z$

CTFT av $\delta \equiv 1$

$$h(t) = \delta(t) + h_0(t) \rightarrow h * x = \delta * x + h_0 * x = x + h_0 * x$$

$$h_0(t) = (\cos t + 2\sin t)u(t) \textit{ Ej integrerbar (CTFT ej def)}$$

$$h_0(t) = (\alpha e^{jt} + \beta e^{-jt})u(t)$$

$$(h_0(t) * x)(t) = \begin{cases} \int_0^t (\alpha e^{j\tau} + \beta e^{-j\tau})e^{-2(t-\tau)}d\tau, t > 0 \\ 0 \end{cases}$$

$$= \alpha(\int_0^t e^{\tau(j+2\tau)}d\tau)e^{-2t} + \beta\int_0^t e^{-\tau(j-2\tau)}e^{-2t}, t > 0$$

$$A = \left[\frac{e^{\tau(j+2)}}{j+2}\right]_0^t = \frac{1}{2+j}(e^{(2+j)t} - 1)$$

$$B = \left[\frac{-e^{\tau(j-2)}}{j-2}\right]_0^t = \frac{1}{2-j}(e^{-(2-j)t} - 1)$$

$$(h_0 * x)(t) = \frac{\alpha}{2+j}(e^{(2+j)t} - 1)e^{-2t}u(t) + \frac{\beta}{2-j}(e^{-(2-j)t} - 1)e^{-2t}u(t)$$

26 Okt/15

$$y = h * x, H(j\omega) = \frac{400}{20 + j\omega)^2}$$
$$x(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(\frac{n\pi t}{L})$$
$$L = \frac{\pi}{10} \qquad y(t) = \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi t}{L}$$

Bestäm $|B_n|$ för de 3 första nollskilda termerna.

Lösning:

$$c_k(y) = H(j\omega_k)c_k(x), \omega_k = \frac{2\pi k}{2L} = \frac{\pi k}{L}$$

Observationer: x reell signal.

$$c_k(x) = \frac{1}{T}\int_0^T x(t)e^{-j\omega_k t}dt \to \bar{x}_k(x) = \frac{1}{T}\int_0^T x(t)e^{j\omega_k t}dt = c_{-k}(x)$$
$$y(t) = \sum_{k=-\infty}^{\infty} c_k(y)e^{j\omega_k t} = [c_0(y) = 0]$$
$$= \sum_{k=1}^{\infty}(c_k(y)e^{j\omega_k t} + c_{-k}(y)e^{-j\omega_k t})$$

$h(t) = 400te^{-20t}u(t)$ tabell värde

$$x(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n}\left(\frac{e^{j\omega_n t} - e^{-j\omega_n t}}{2j}\right)$$

Tiden gick ut, så föreläsaren hänvisar till kurshemsidan